Solving with Factoring and the Quadratic Formula

One of the most important skills in algebra is knowing when an equation requires solving and how to go about doing so.

The purpose of this worksheet is:

- To familiarize the user with the types of equations that require factoring or the quadratic formula to arrive at a solution.
- To provide practice in distiniguishing between single-variable-form equations and multiple-variable form equations.
- To provide practice in determining whether or not it is practical to factor a multiple-variable form equation; and, if not, how to properly apply the quadratic formula.

Note: This topic is largely a part of the foundation of mathematics and has been studied for hundreds of years. This worksheet will not provide all answers. But the worksheet will provide a foundation for consolidating a number of skills the student should have mastered by the completion of Algebra Two.

THE SINGLE-VARIABLE-FORM EQUATION.

A **single-variable-form equation** is an equation in which the variable for which you are trying to solve takes on only one form in any term, for example: In the equation $5x^3 - 7 = 3 - 2x^3$, there is only one form of *x*: x^3. But in the equation $3\log(x^2) - 7x^2 = 4$, there are two forms of *x*: $\log(x^2)$ is one form and x^2 is the other form.

What is special about single-variable-form equations? These equations can be solved by isolating the variable – this is a lot simpler than factoring or any other method.

Example: Solve for *x* in the equation $5x^3 - 7 = 3 - 2x^3$.

Solution: $5x^3 - 7 = 3 - 2x^3$

$$3x^3 = 10$$

$$x^3 = \frac{10}{3}$$

$$x = \left(\frac{10}{3}\right)^{\frac{1}{3}}$$

Here's some practice: Determine which of the following equations are single-variable form in *x* and which are **multiple-variable-form** in *x*. Solve for *x* in the equations that are single-variable-form.

1. $9x^3 - 7 = x$
2. $5x^7 - 8 = 4(x^7 + 3) - x^7$
3. $\pi x + 9 + c = 5x + c^2$
4. $x^2 - 17 = \dfrac{1}{x^2}$

To get you started:

1. This equation is not single-variable-form in *x*. There are two forms of *x*: x^3 and x.
2. This equation is single-variable-form.
3. This equation is single-variable-form.
4. This equation is multiple-variable form: x^2 and $\dfrac{1}{x^2}$.

THE QUADRATIC FORMULA

$$\boxed{\text{If } ax^2+bx+c=0, \text{ then } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.}$$

You need to understand that this is a *formula* and the formula can be applied to any equation matching the form given in the formula.

Example: Solve the equation: $4w^6-8=3w^3$ using the quadratic formula.

The equation is not in the proper form and needs to be rewritten.

$$4\left(w^3\right)^2-3\left(w^3\right)-8=0$$

Now, we can identify the parts of the formula with the parts of the equation:

- The x represents the parenthetical expression $\left(w^3\right)$.
- The a represents the "quadratic" coefficient 4.
- The b represents the "linear" coefficient -3.
- The c represents the "constant term" -8.

Substitution in the quadratic formula gives us the equation:

$$w^3=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-8)}}{2(4)}$$

Simplifying yields:

$$w^3=\frac{3\pm\sqrt{9+128}}{8}$$

$$w^3=\frac{3\pm\sqrt{137}}{8}$$

Solving for w produces:

$$w=\sqrt[3]{\frac{3\pm\sqrt{137}}{8}}$$

$$w=\frac{\sqrt[3]{3+\sqrt{137}}}{2},\ \frac{\sqrt[3]{3-\sqrt{137}}}{2}$$

Note: The Quadratic Formula should not be used to avoid factoring. Factoring is a valuable skill and will assist with problems for which the Quadratic Formula would be inappropriate. Always try factoring first.

Here are some problems that should be worked using the Quadratic Formula. Solve for u in each of the following equations.

5. $-3u^2-10u=2$ 6. $5u^2+9u-k=0$ 7. $7\left(2u\right)^2-4=-\left(2u\right)$ 8. $\left(5^u\right)^2=4\left(5^u\right)-2$ 9. $\left(u-7\right)+4\left(u-7\right)^2-3=0$

To get you started here are the rewritings and substitutions you will have to make to use the Quadratic Formula:

5. $-3u^2-10u-2=0$
   $x \Rightarrow u$
   $a \Rightarrow -3$
   $b \Rightarrow -10$
   $c \Rightarrow -2$

6. $5u^2+9u-k=0$
   $x \Rightarrow u$
   $a \Rightarrow 5$
   $b \Rightarrow 9$
   $c \Rightarrow -k$

7. $7\left(2u\right)^2+(2u)-4=0$
   $x \Rightarrow \left(2u\right)$
   $a \Rightarrow 7$
   $b \Rightarrow 1$
   $c \Rightarrow -4$

8. $\left(5^u\right)^2-4\left(5^u\right)+2=0$
   $x \Rightarrow \left(5^u\right)$
   $a \Rightarrow 1$
   $b \Rightarrow -4$
   $c \Rightarrow 2$

9. $4\left(u-7\right)^2+\left(u-7\right)-3=0$
   $x \Rightarrow \left(u-7\right)$
   $a \Rightarrow 4$
   $b \Rightarrow 1$
   $c \Rightarrow -3$

FACTORING

Once you have decided you cannot solve by isolating the variable, you should explore factoring. These are the key steps:

- The equation should be rewritten (if necessary) so that:
  - The equation is zeroed.
  - The equation is in descending order.
  - The equation is reduced as far as possible.
  - The lead term is positive.
- Monomial factors should be factored out first!
  - (This is very important. Forgetting to do this is one of the most common mistakes of students; and usually results in equations that are improperly solved.)
- Binomial-form factors should be factored out next.

Note: Remember that, in general, you may not reduce an equation by a variable expression.

Example: Solve for x in the equation $140x^2 - 60x^3 = 80x$

zero the equation and write in descending order	$-60x^3 + 140x^2 - 80x = 0$		
reduce the equation by 20	$-3x^3 + 7x^2 - 4x = 0$		
change signs so that lead sign is negative (this could be accomplished by reducing by -20 in the previous step)	$3x^3 - 7x^2 + 4x = 0$		
factor out monomial factors!	$x\left(3x^2 - 7x + 4\right) = 0$		
factor out binomials	$x(x-1)(3x-4) = 0$		
branch and solve	$x = 0$	$x - 1 = 0$ $x = 1$	$3x - 4 = 0$ $3x = 4$ $x = \frac{4}{3}$
consolidate and simplify the solutions	$x = 0, 1, -1\frac{1}{4}$		

Practice Problems. Solve for x using the best method of solving.

1. $5x^2 + 4 = 3x^2 + 54$
2. $5x^3 - 15x^2 = 0$
3. $-14x^2 + 10 = -2$
4. $(2x+3)^2 = 5x + 9$
5. $22x^3 + 77x = 165x^2$
6. $e^x\left(17 - 2e^x\right) = 21$
7. $x\left(6\pi x + 2\right) = 12$
8. $a^2 x + e^2 = ax$
9. $5\left(x^2 - 4\right)^2 - 7\left(x^2 - 4\right) - 6 = 0$
10. $3x^2 - 29x^3 = -6x^4$
11. $\frac{3x^2}{4} - 4x + 1\frac{1}{4} = 0$
12. $\frac{1}{3}x^2 + \frac{5}{6}x = \frac{1}{4}$