

## COMPLETING THE SQUARE (IN PLACE) OF TRINOMIAL QUADRATIC EXPRESSIONS

To complete the square, there are three parts: Preparation, Completing the Square, and Clean-up. All together, there are five steps.

Prepare the expression

- Write the expression in descending order then isolate the variables terms in square brackets.
- Factor out the lead coefficient of the expression in the square brackets.

Complete the square

- Complete the square of the parenthetical expression as follows:  
(variable plus one-half the coefficient of the linear term)<sup>2</sup> minus the square of half the linear term.

Clean up the expression

- Distribute over the square brackets.
- Combine like terms.

Example 1

Complete the square:  $4w^2 - 24w + 1$

Step 1: Isolate	$[4w^2 - 24w] + 1$
Step 2: Factor	$4[w^2 - 6w] + 1$
Step 3: Complete the square	$4[(w-3)^2 - 9] + 1$
Step 4: Distribute	$4(w-3)^2 - 36 + 1$
Step 5: Combine like terms	$4(w-3)^2 - 35$

Example 2

Complete the square:  $-x^2 + 4x$

Step 1:	$[-x^2 + 4x]$
Step 2:	$-[x^2 - 4x]$
Step 3:	$-[(x-2)^2 - 4]$
Step 4:	$-(x-2)^2 + 4$
Step 5: (not needed)	

Example 3

Complete the square:  $3y^2 + 3y + 5$

Step 1:	$[3y^2 + 3y] + 5$
Step 2:	$3[y^2 + y] + 5$
Step 3:	$3[(y + \frac{1}{2})^2 - \frac{1}{4}] + 5$
Step 4:	$3(y + \frac{1}{2})^2 - \frac{3}{4} + 5$
Step 5:	$3(y + \frac{1}{2})^2 + 4\frac{1}{4}$

Example 4

Complete the square:  $-5z^2 - 12z - 7$

Step 1:	$[-5z^2 - 12z] - 7$
Step 2:	$-5[z^2 + \frac{12}{5}z] - 7$
Step 3:	$-5[(z + \frac{6}{5})^2 - \frac{36}{25}] - 7$
Step 4:	$-5(z + \frac{6}{5})^2 + \frac{36}{5} - 7$
Step 5:	$-5(z + \frac{6}{5})^2 + \frac{1}{5}$

Complete the square of each of the following.

1.  $2a^2 + 12a + 2$

2.  $2b^2 - 8b + 2$

3.  $-2c^2 + 4c + 3$

4.  $-3d^2 - 18d - 2$

5.  $-g^2 + 10g - 24$

6.  $h^2 + 8h + 17$

7.  $5j^2 - 20j + 20$

8.  $-3k^2 + 30k$

9.  $-4m^2 - 16m$

10.  $n^2 + 6$

11.  $-7p^2 - 42p - 63$

12.  $8q^2 + 160q$

13.  $-4r^2 + 32r + 100\frac{1}{3}$

14.  $-s^2 - 18s - 90.2$

15.  $t^2 - 14t + 49$

16.  $2u^2 + 48u + 143\frac{2}{5}$

17.  $-v^2 + 7$

18.  $-6w^2 + 12w$

19.  $3x^2 + 120x + 1200$

20.  $-9y^2 - 180y + 1.82$

21.  $z^2 + 9z + 20$

22.  $2A^2 - 14A - 3$

23.  $7B^2 + 7B$

24.  $-3C^2 + 15C - 2\frac{1}{4}$

25.  $-2D^2 - 18D + 6\frac{1}{2}$

26.  $E^2 - 11E + 30\frac{1}{4}$

27.  $-F^2 + 3\frac{2}{7}$

28.  $-G^2 - 7G$

29.  $3H^2 + 27H - 2$

30.  $2J^2 - 9J + 5$

31.  $-3K^2 - 8K - 1$

32.  $-L^2 + 28L$

33.  $\frac{1}{3}M^2 + \frac{4}{3}M + 1$

34.  $\frac{2}{5}N^2 - N$

35.  $-\frac{1}{6}P^2 + 2P + 30$

36.  $\frac{2}{3}Q^2 - \frac{1}{2}Q + 2$

37.  $\frac{1}{10}R^2 + \frac{6}{5}R$

38.  $\frac{-2}{7}S^2 - \frac{3}{14}S + 1$

39.  $\frac{5}{2}T^2 + 10T + 3$

40.  $2U^2 - \frac{7}{3}U - 5$

**Answers:**

1.  $2(a+3)^2 - 16$

2.  $2(b-2)^2 - 6$

3.  $-2(c-1)^2 + 5$

4.  $-3(d+3)^2 + 25$

5.  $-(g-5)^2 + 1$

6.  $(h+4)^2 + 1$

7.  $5(j-2)^2$

8.  $-3(k-5)^2 + 75$

9.  $-4(m+2)^2 + 16$

10.  $n^2 + 6$

11.  $-7(p+3)^2$

12.  $8(q+10)^2 - 800$

13.  $-4(r-4)^2 + 164\frac{1}{3}$

14.  $-(s+9)^2 - 9.2$

15.  $(t-7)^2$

16.  $2(u+12)^2 - 144\frac{3}{5}$

17.  $-v^2 + 7$

18.  $-6(w-1)^2 + 6$

19.  $3(x+20)^2$

20.  $-9(y+10)^2 + 901.82$

21.  $(z+4\frac{1}{2})^2 - \frac{1}{4}$

22.  $2(A-3\frac{1}{2})^2 - 27\frac{1}{2}$

23.  $7(B+\frac{1}{2})^2 - 1\frac{3}{4}$

24.  $-3(C-2\frac{1}{2})^2 + 16\frac{1}{2}$

25.  $-2(D+4\frac{1}{2})^2 + 47$

26.  $(E-5\frac{1}{2})^2$

27.  $-F^2 + 3\frac{2}{7}$

28.  $-(G+3\frac{1}{2})^2 + 10\frac{1}{4}$

29.  $3(H+4\frac{1}{2})^2 - 62\frac{3}{4}$

30.  $2(J-2\frac{1}{4})^2 - 5\frac{1}{8}$

31.  $-3(K+1\frac{1}{3})^2 + 4\frac{1}{3}$

32.  $-(L-14)^2 + 196$

33.  $\frac{1}{3}(M+2)^2 - \frac{1}{3}$

34.  $\frac{2}{5}(N-1\frac{1}{4})^2 - \frac{5}{8}$

35.  $-\frac{1}{6}(P-6)^2 + 6$

36.  $\frac{2}{3}(Q-\frac{3}{8})^2$

37.  $\frac{1}{10}(R+6)^2 - 3\frac{3}{5}$

38.  $\frac{-2}{7}(S+\frac{3}{8})^2 - 7$

39.  $\frac{5}{2}(T+2)^2 - 7$

40.  $2(U-\frac{7}{12})^2 - 5\frac{49}{72}$